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*Title:* Tomographic Reconstruction for Cosmic Ray Muon Radiography (U)

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October 16-22, 2004

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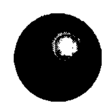
# Tomographic Reconstruction for Cosmic Ray Muon Radiography


Larry Schultz

Los Alamos National Laboratory


October 19, 2004

*2004 IEEE NSS/MIC/SNPS & RTSD*



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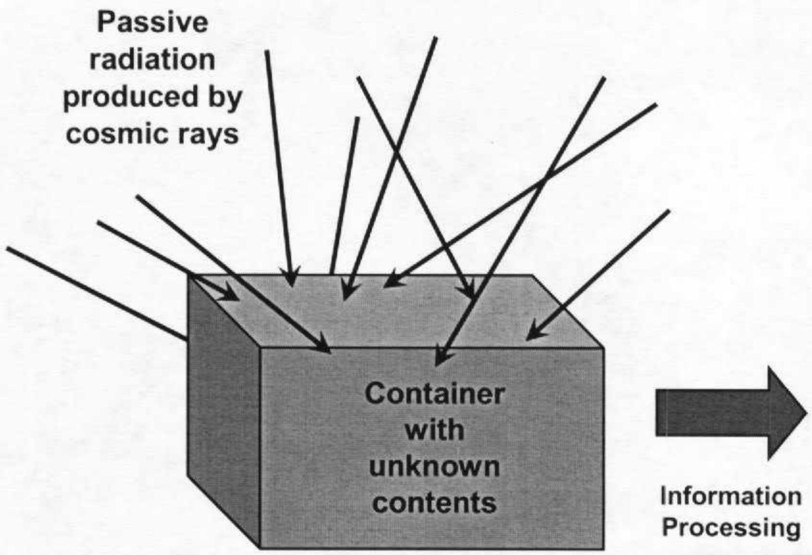
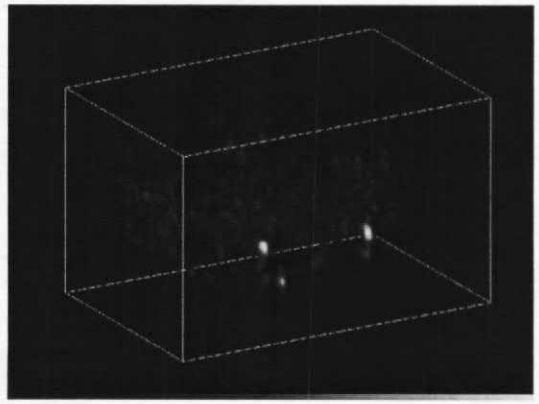
# The topic of this talk...




Radiograph revealing contents in a minute or so without application of any artificial radiation.



# Collaborators

- **Los Alamos National Laboratory**
  - C.L. Morris, W.C. Priedhorsky, T.J. Asaki, K.N. Borozdin, R. Chartrand, J.J. Gomez, N.W. Hengartner, G.E. Hogan, J.P. Lestone, A. Saunders, L.J. Schultz, R.C. Schirato, K.R. Vixie.
  - M.E. Teasdale, M.Q. Cannon, J.J. Medina (students)
- **Portland State University**
  - A. Fraser
- **University of South Carolina**
  - G. Blanpied
- **Washington University in St. Louis**
  - J. Katz

# Agenda

- Background & Concept
- Experimental Proof of Principle & Simulation
- Point of Closest Approach Reconstruction
- Maximum Likelihood Reconstruction
- Summary / Future Work

# Background & Concept

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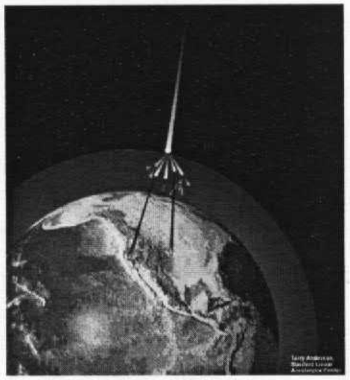
# Cosmic Ray Muons

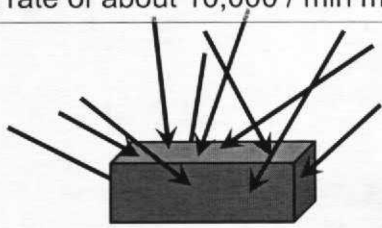
Produced from primary cosmic rays in the atmosphere.



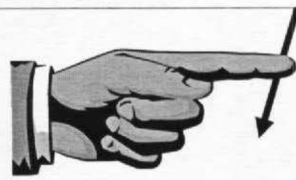
Many muons can penetrate several meters of rock.

© National Geographic, 1963

Muons arrive from upper hemisphere at a rate of about 10,000 / $\text{min}\cdot\text{m}^2$.



That's about one through your fingernail per minute.



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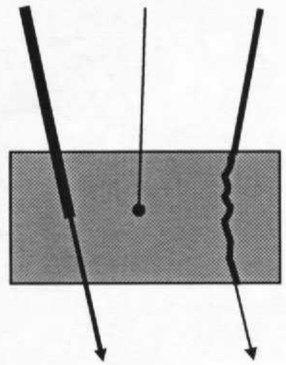
# How Muons Interact with Material

- Energy Loss
- Range Out
- Multiple Scattering

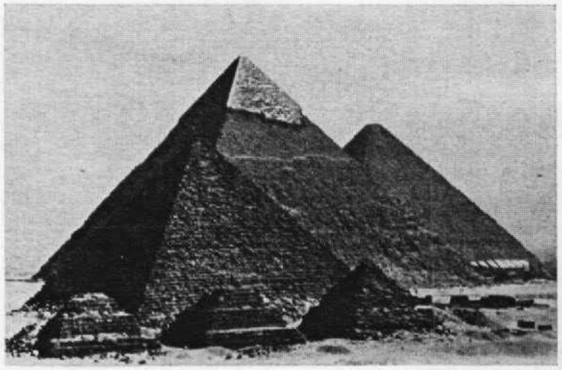


*These interaction modes depend material properties, and so represent potential material identification information sources.*

# Differential Attenuation Radiography

*Searching for Hidden Chambers in Pyramids*

Fig. 1 (top right). The pyramids at Giza. From left to right, the Third Pyramid of Mycerinus, the Second Pyramid of Chephren, the Great Pyramid of Cheops. [© National Geographic Society]



Luis Alvarez, et. al.
*Science* **167**, 832 (1970)

Arturo Menchaca, et. al.
current effort, see

***http://www.msnbc.msn.com/id/4540266/***

*Predicting Volcanic Eruptions*

Tanaka, Nagamine, et. al.
*Nuclear Instruments and Methods A*
**507**:3, 657 (2003)

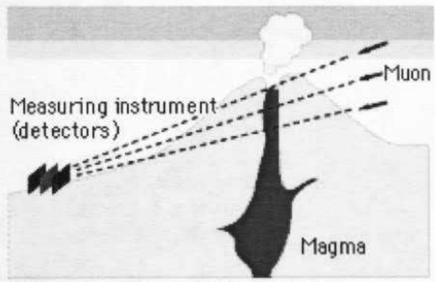



Figure 4: Analyzing the internal structure of a volcanic zone using muons

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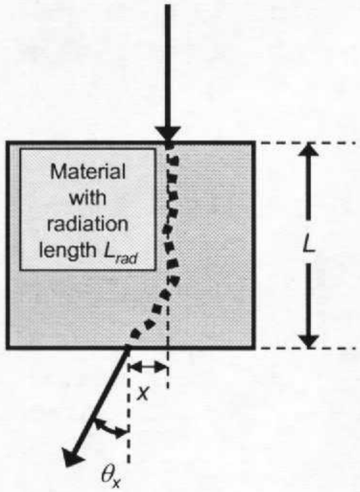
# Multiple Scattering

The particle experiences many small angle scatters due to E.M. interactions with material atoms.

Charged particle with momentum *p.*




Particle is scattered and displaced from the original track.

Distribution of scattering angle is approximately Gaussian:

$$f_{\theta_x}(\theta_x) \cong \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{\frac{\theta_x^2}{2\sigma_\theta^2}}$$

Standard deviation is related to the material:

$$\sigma_\theta \cong \frac{15}{p}\sqrt{\frac{L}{L_{rad}}}$$

The *radiation length* $L_0$ is a characteristic property of material that generally DECREASES with INCREASING material $Z$ number.

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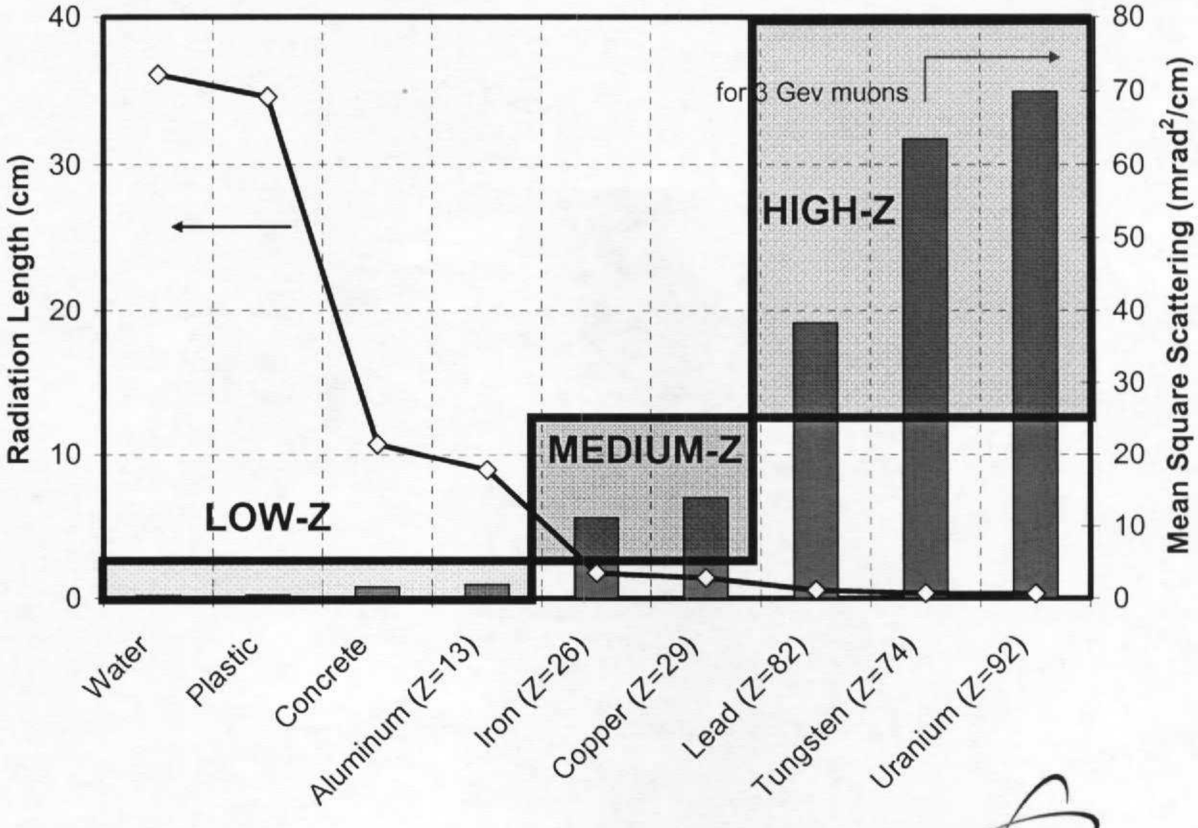
# Scattering is Material Dependent

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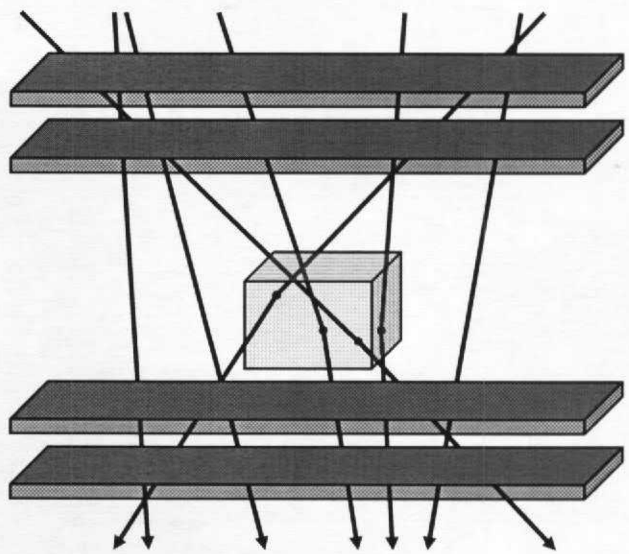
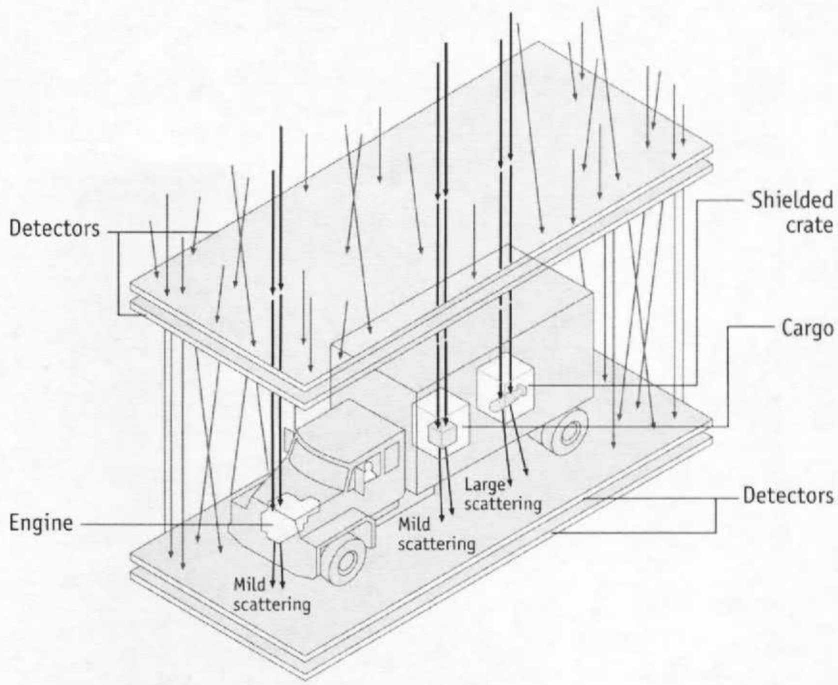
# The Basic Concept

- Track individual muons (possible due to modest event rate).
- Track muons into and out of an object volume.
- Determine scattering angle of each muon.
- Infer Z-level of material within volume from data provided by many muons.
- New tomographic reconstruction algorithms are required.



# Muon Radiography for Detection of Contraband Special Nuclear Material

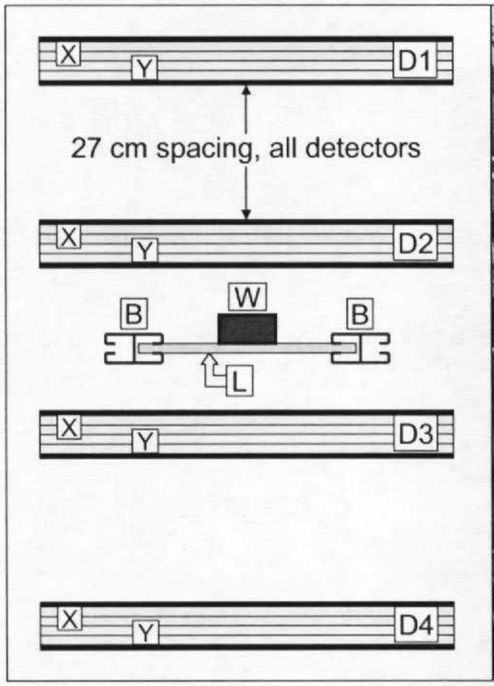
- No artificial radiological dose.
- No artificial source required.
- Low-cost muon detectors have been used for decades.
- 3D reconstruction enabled by multi-angle "illumination."
- The heavier the shield the better.
- Existing methods don't work well for detecting uranium.

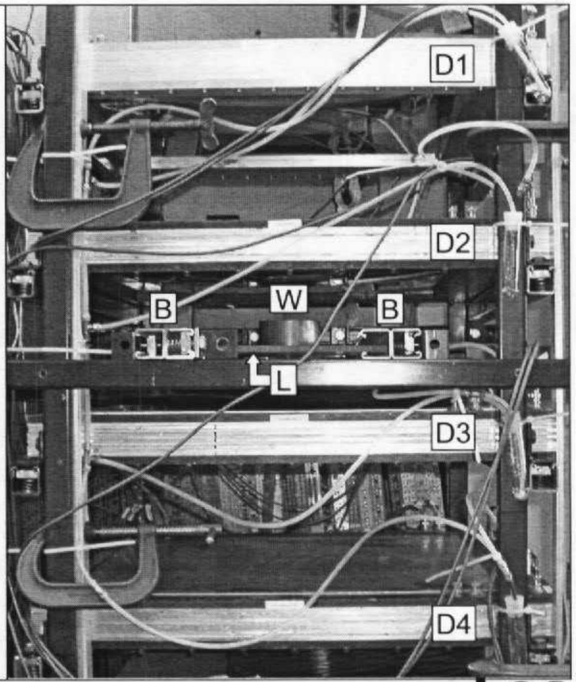


# Experimental Proof of Principle & Simulation

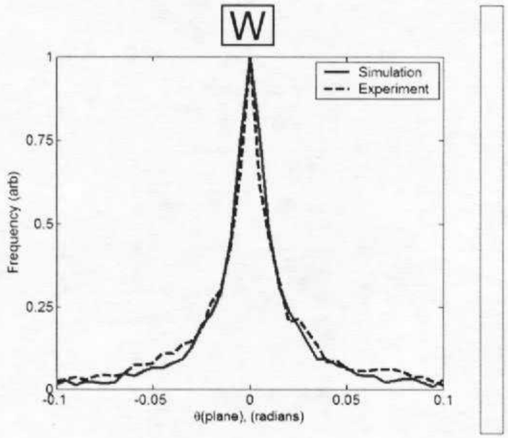
# Experimental Prototype

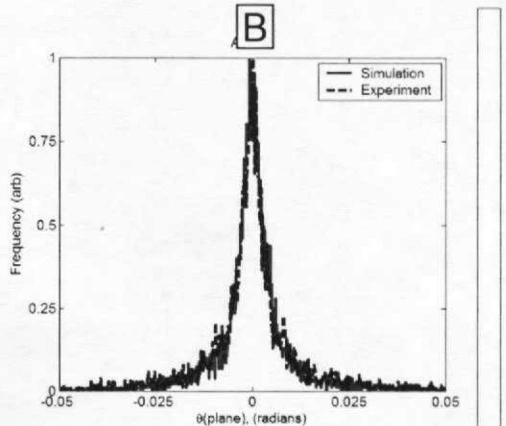
- D1-D4 – Muon detectors (wire chambers, measure X/Y position)
- B – "Unistrut" beams
- L – Lexan plate to hold objects
- W – Tungsten cylinder (5.5 cm radius, 5.8 cm height)

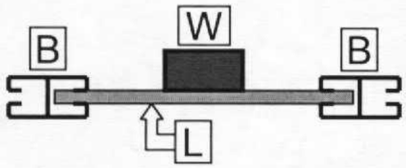


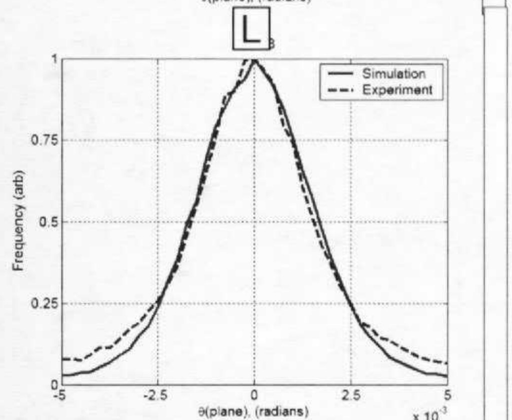



# Scattering Histograms

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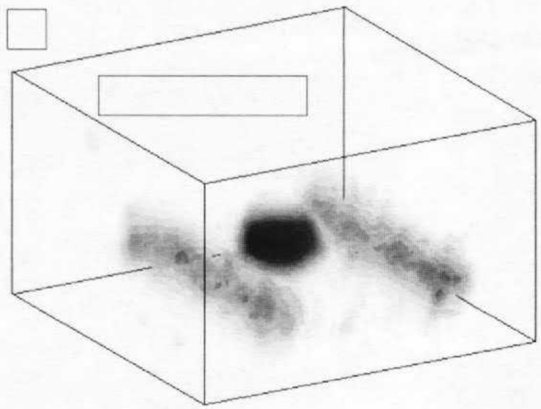
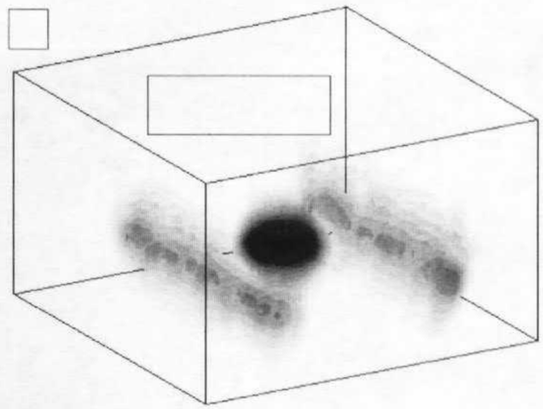
# Reconstructed Images

Experiment



Simulation

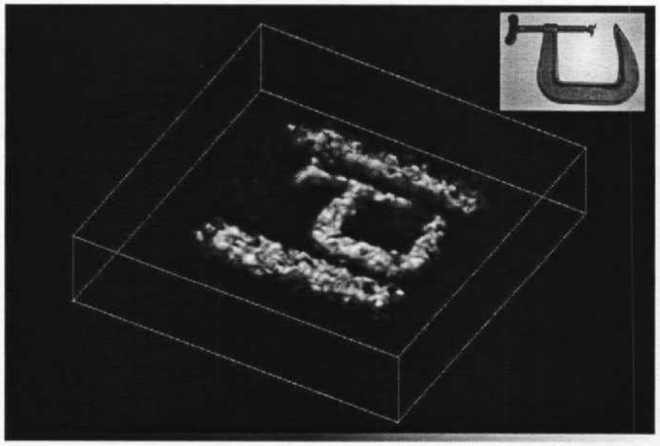


*More on how the images were made later.*

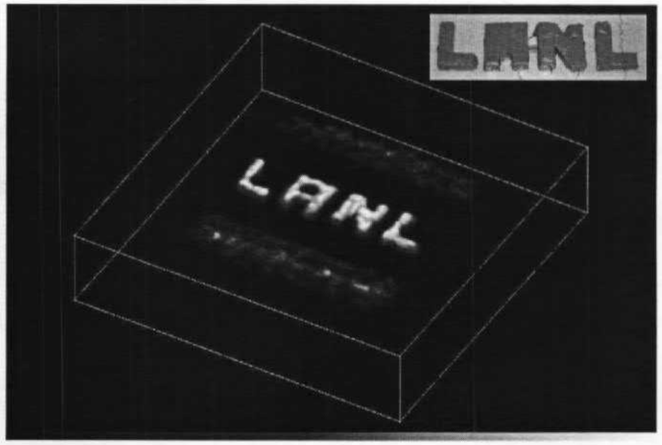
*Standalone Monte Carlo muon transport simulation developed.*
*GEANT 4 based simulation has also been tested.*

# More Experimental Radiographs

A Steel C-Clamp

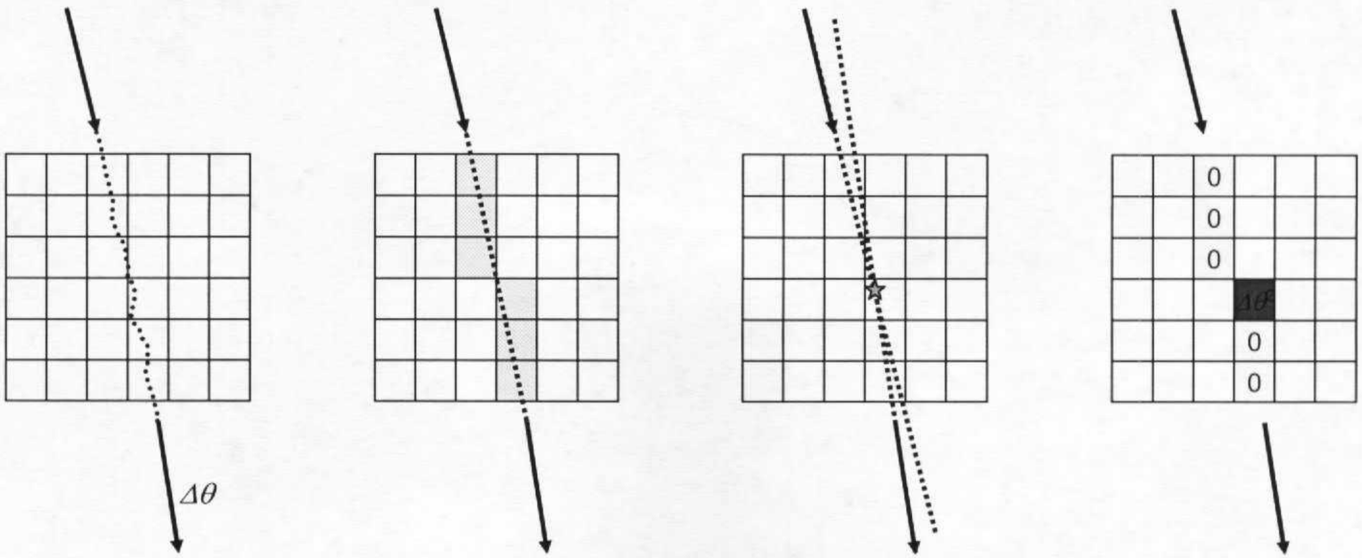


LANL of Lead



# Point of Closest Approach Reconstruction Algorithm

# PoCA Reconstruction Algorithm




A ray takes a stochastic, path and emerges with an aggregate scattering angle.

Estimate the ray path and identify pixels that "influenced" the ray.

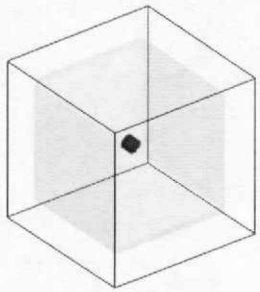
Find point of closest approach (PoCA). Assume all scattering occurred in that pixel.

Assign squared scattering angle to PoCA pixel, 0 to other candidate pixels.

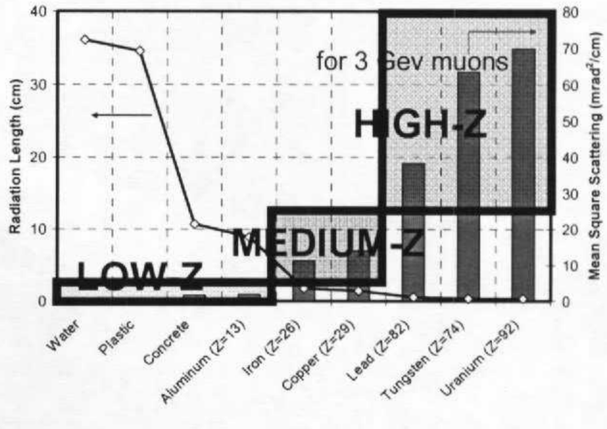
*Taking the mean signal assigned to each pixel over many muons results in an a reconstruction of material scattering tendency.*

# PoCA works well for...

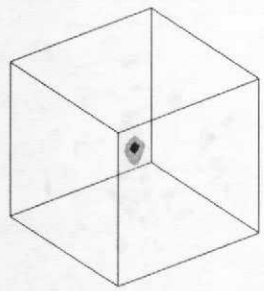
Scenes with relatively small isolated objects



Simulated scene
1x1x1 $m^3$ Fe box (3 mm wall thickness)
4 cm radius U sphere in center




PoCA Reconstruction
~ 1 minute of simulated exposure



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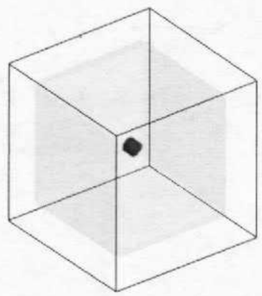
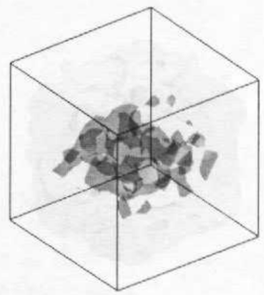
# PoCA works less well for...

Scenes with large, distributed, or multiple objects



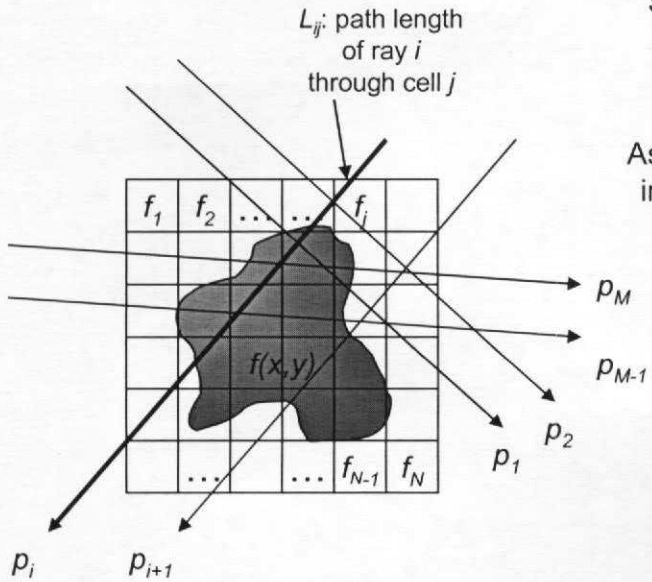
Simulated scene
1x1x1 m3 Fe box (3 mm wall thickness)
now filled with solid Aluminum
and 4 cm radius U sphere in center



PoCA Reconstruction
~ 1 minute of simulated exposure

# Maximum Likelihood Reconstruction

# "Traditional" Iterative Tomographic Reconstruction




Start with a discrete object grid with *N* elements.

A ray passes through the grid.

Assume that ray values may be represented by line integrals of the object function along the ray path.

Or, in discrete form, the raysum for ray *i* is

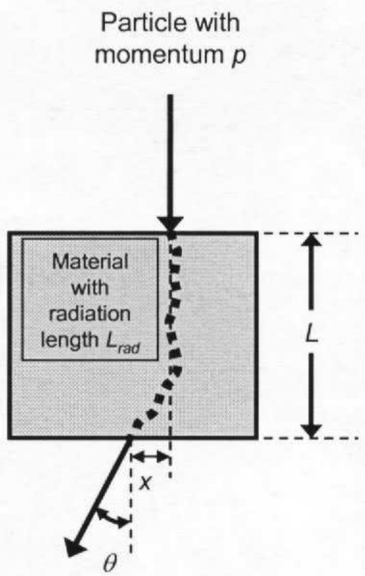
$$p_i = \sum_{j=1}^{N} L_{ij} f_j + n_i$$

Pass *M* rays through the grid.

$$\mathbf{p} = \mathbf{Lf} + \mathbf{n}$$

Solve this system iteratively.

# Scattering Density




$$f_\theta(\theta) \cong \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{\theta^2}{2\sigma_\theta^2}}$$

$$\sigma_\theta = \frac{15}{p}\sqrt{\frac{L}{L_{rad}}}$$

Establish a nominal particle momentum $p_0$

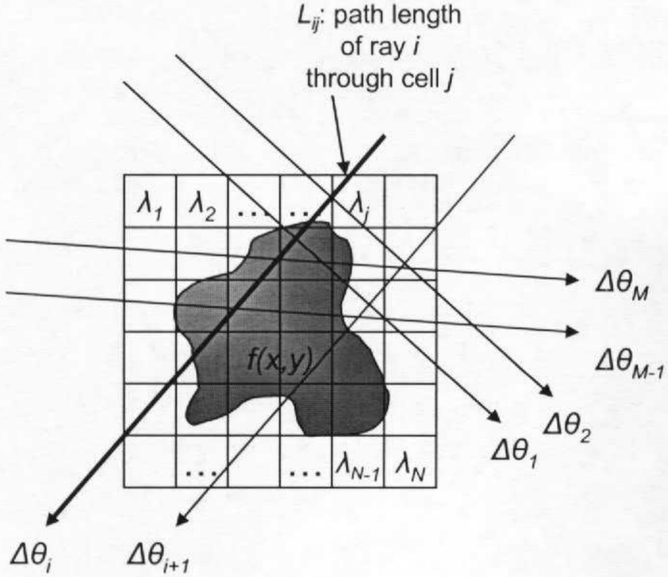
and define the *scattering density* of a material as:

$$\lambda_{material} = \left(\frac{15}{p_0}\right)^2 \frac{1}{L_{rad,material}}$$

*Scattering density is the mean square scattering of nominal momentum muons per unit length of a material.*

$$\sigma_\theta^2 = \left(\frac{p_0}{p}\right)^2 L\lambda$$

# When Scattering is the Signal




Estimated VARIANCE of scattering expected for a nominal muon making the path of ray *i* is expressed as:

$$\hat{v}_i \equiv \hat{\sigma}^2_{\Delta\theta} = \sum_{j=1}^{N} L_{ij}\hat{\lambda}_j$$

CONDITIONAL PROBABILITY of the scattering of ray *i* given the scattering density estimate is:

$$P\left(\Delta\theta_i \middle| \hat{v}_i\right) = \frac{1}{\sqrt{2\pi\hat{v}_i}} \exp\left(-\frac{\Delta\theta_i^2}{2\hat{v}_i}\right)$$

The scattering of each ray is uncorrelated with the scattering of any other ray, so the probability of the entire dataset over *M* rays is:

$$P\left(\mathbf{\Delta\theta} \middle| \hat{\mathbf{v}}\right) = \prod_{i=1}^{M} P\left(\Delta\theta_i \middle| \hat{v}_i\right)$$

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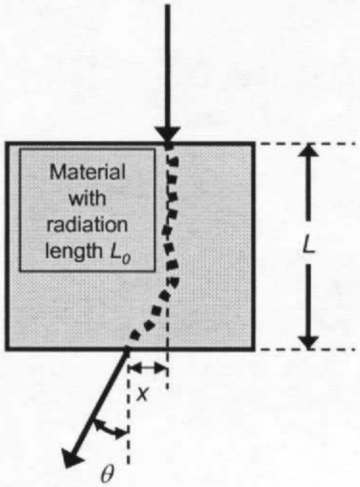
# Incorporating Displacement




Displacement is encompassed in multiple scattering theory.

Bend angle and displacement may be described by a correlated 2D Gaussian distribution.

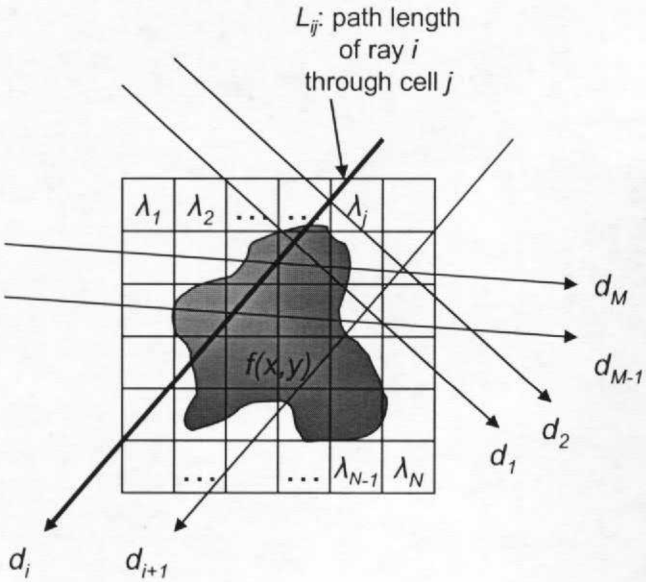
$$f(\theta,x)=\frac{1}{2\pi\sqrt{|\Sigma|}}\exp\left[-\frac{1}{2}[\theta \quad x]^T\Sigma^{-1}[\theta \quad x]\right]$$

$$\Sigma=\begin{bmatrix}\sigma_\theta^2 & \sigma_{\theta x}\\ \sigma_{\theta x} & \sigma_x^2\end{bmatrix}$$

$$\sigma_x^2=\frac{L^2}{3}\sigma_\theta^2 \qquad \sigma_{\theta x}=\frac{L}{2}\sigma_\theta^2$$

$$\sigma_\theta^2=L\lambda \qquad \sigma_x^2=\frac{L^3}{3}\lambda \qquad \sigma_{\theta x}=\frac{L^2}{2}\lambda$$

# Using Scattering and Displacement




Now each ray carries the signal:

$$\mathbf{d_i} = \begin{bmatrix} \Delta\theta_i \\ \Delta x_i \end{bmatrix}$$

Expressions can be derived for the covariance matrix as a function of ray paths and scattering density estimates:

$$\hat{\mathbf{\Sigma}}_i = \begin{bmatrix} \hat{v}_{\theta i} & \hat{s}_{\theta x i} \\ \hat{s}_{\theta x i} & \hat{v}_{x i} \end{bmatrix} = \begin{bmatrix} \mathbf{W_{\theta,i}}\hat{\boldsymbol{\lambda}} & \mathbf{W_{\theta x,i}}\hat{\boldsymbol{\lambda}} \\ \mathbf{W_{\theta x,i}}\hat{\boldsymbol{\lambda}} & \mathbf{W_{x,i}}\hat{\boldsymbol{\lambda}} \end{bmatrix}$$

And the probability of the measured signal $\mathbf{d_i}$ given the covariance matrix estimate is:

$$P\left(\mathbf{d}_i \middle| \hat{\mathbf{\Sigma}}_i\right) = \frac{1}{2\pi\left|\hat{\mathbf{\Sigma}}_i\right|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{d}_i^{\mathbf{T}}\hat{\mathbf{\Sigma}}_i^{\mathbf{-1}}\mathbf{d}_i\right)$$

# Maximizing Likelihood

Define the optimal estimate as the estimate which maximizes the conditional probability:

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} P\left(\hat{\boldsymbol{\lambda}} \middle| \mathbf{d}\right)$$

Expanding the right side using Bayes Law:

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} \frac{P\left(\mathbf{d} \middle| \hat{\boldsymbol{\lambda}}\right) P\left(\hat{\boldsymbol{\lambda}}\right)}{P(\mathbf{d})}$$

The denominator term is a constant, and if we assume that any object makeup is equally probable:

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} P\left(\mathbf{d} \middle| \hat{\boldsymbol{\lambda}}\right)$$

# Maximizing LOG Likelihood

Maximizing the log of the likelihood is equivalent to maximizing likelihood:

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} \ln\left[P\left(\mathbf{d}\middle|\hat{\boldsymbol{\lambda}}\right)\right]$$

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} \sum_{i=1}^{M} \ln\left[P\left(\mathbf{d}_{\mathbf{i}}\middle|\hat{\boldsymbol{\Sigma}}_{\mathbf{i}}\right)\right]$$

Recall that the ray probabilities are Gaussian:

$$\hat{\boldsymbol{\lambda}}^* = \arg\max_{\hat{\lambda}} \sum_{i=1}^{M} \left[-\ln(2\pi) - \frac{1}{2}\ln\left(\left|\hat{\boldsymbol{\Sigma}}_i\right|\right) - \frac{1}{2}\mathbf{d}_i^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}_i^{-1}\mathbf{d}_i\right]$$

$$\hat{\boldsymbol{\lambda}}^* = \arg\min_{\hat{\lambda}} \sum_{i=1}^{M} \left[\ln\left(\left|\hat{\boldsymbol{\Sigma}}_i\right|\right) + \mathbf{d}_i^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}_i^{-1}\mathbf{d}_i\right]$$

# The Minimization Problem

We now have a cost function for a minimization problem:

$$F\left(\hat{\boldsymbol{\lambda}}\right)=\sum_{i=1}^{M}\left[\ln\left(\left|\hat{\boldsymbol{\Sigma}}_i\right|\right)+\mathbf{d}_i^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}_i^{-1}\mathbf{d}_i\right]$$

However, we need to constrain scattering densities to positive values, in fact, to values at least representative of air.

Hence we define the *constrained* minimization problem:

$$\hat{\boldsymbol{\lambda}}^* = \underset{\hat{\boldsymbol{\lambda}}}{\arg\min}\, F\left(\hat{\boldsymbol{\lambda}}\right) \text{ such that } \lambda_j > \lambda_{air} \text{ for all } j$$

# Finding a Solution

- **Constrained optimization via trust region based Newton type method**
  - Analytical expressions for Jacobian and Hessian matrices can be developed.
  - Works well for scenes with small number of parameters and rays (e.g., around 1000 voxels, a few thousand rays on a desktop PC).
  - Computation and storage of Hessian matrix limits problem size.
- **Expectation Minimization (EM) Algorithm**
  - Slower convergence than Newton type method.
  - Can handle larger problems (e.g., 100K+ voxels, 100K+ rays).
  - Parallelizable.

# Extensions

## Measurement Error

- Finite detector resolution generates uncertainty in bend angle and displacement measurements.
- Error may be modeled and included in the statistical model.
- For ray *i*

$$\sigma_{\theta,i}^2 = \mathbf{W}_{\theta i}\hat{\boldsymbol{\lambda}} + E_{\Delta\theta}^2$$

$$\sigma_{\theta x,i} = \mathbf{W}_{\theta x i}\hat{\boldsymbol{\lambda}} + E_{\Delta\theta x}$$

$$\sigma_{x,i}^2 = \mathbf{W}_{xi}\hat{\boldsymbol{\lambda}} + E_{\Delta x}^2$$

## Muon Momentum Spread

- Real cosmic ray muons vary in momentum
- If momentum estimates are available then they can be incorporated:
- For ray *i*

$$F_{pi} = M_{2p}\left(\frac{p_0}{\hat{p}_i}\right)^2$$

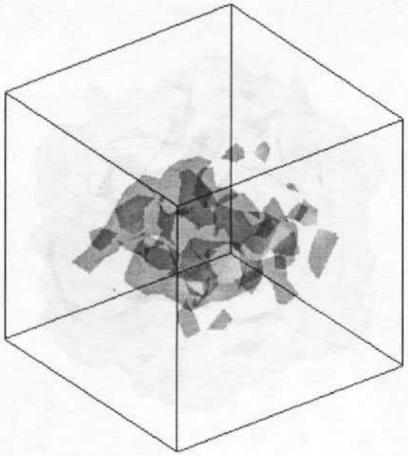
$$\sigma_{\theta,i}^2 = F_{p,i}\mathbf{W}_{\theta i}\hat{\boldsymbol{\lambda}} + E_{\Delta\theta}^2$$

$$\sigma_{\theta x,i} = F_{p,i}\mathbf{W}_{\theta x i}\hat{\boldsymbol{\lambda}} + E_{\Delta\theta x}$$

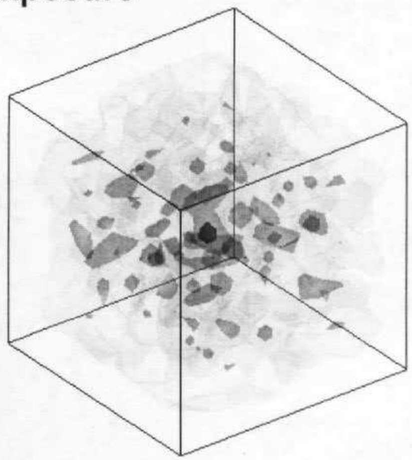
$$\sigma_{x,i}^2 = F_{p,i}\mathbf{W}_{xi}\hat{\boldsymbol{\lambda}} + E_{\Delta x}^2$$

# ML/EM Reconstruction Results

**Previous scene**
**U sphere buried in Al**
**1 minute simulated exposure**



**PoCA Reconstruction**



**ML/EM Reconstruction**

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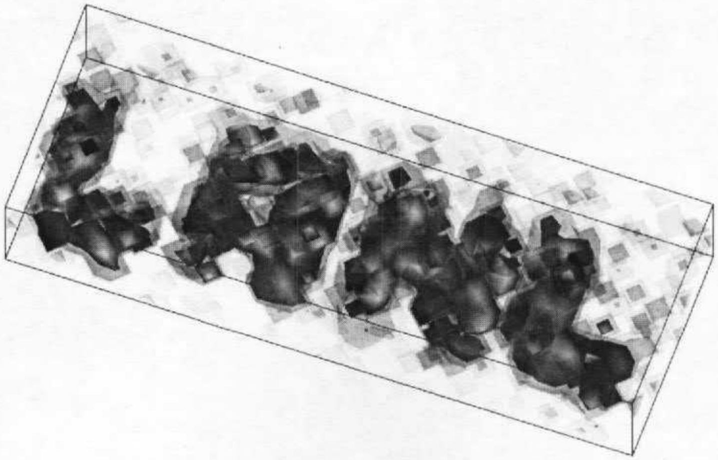
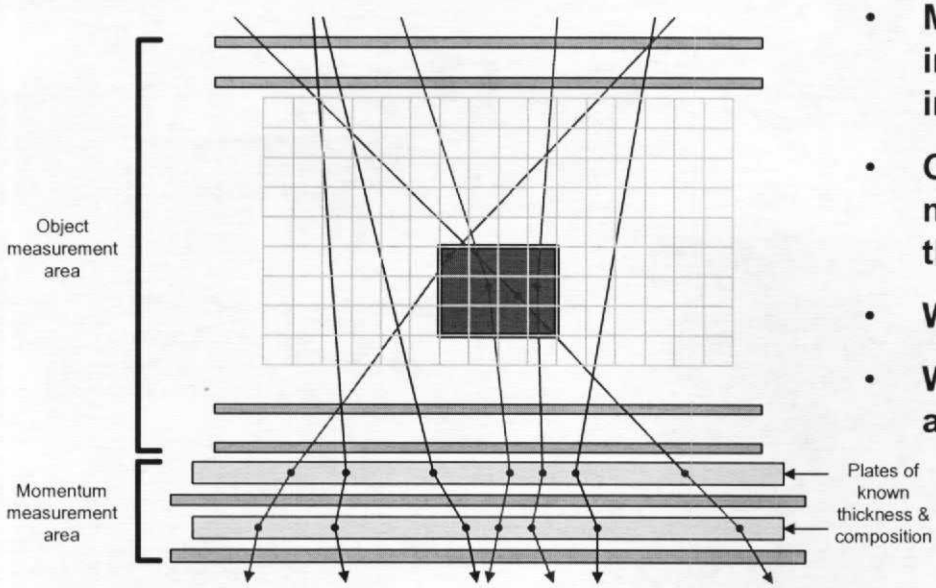
# ML/EM on Experimental Data



A good start but a bit messy.
More work is needed to understand causes.

# Momentum Estimation




- **Measuring particle momentum increases confidence in material inference.**
- **One method is to estimate momentum from scattering through known material.**
- **With 2 plates $\Delta p/p$ is about 50%.**
- **With *N* measurements $\Delta p/p$ approaches:**

$$\sqrt{\frac{1}{2N}}$$

# Summary / Future Work

- **Cosmic Ray Muon Radiography is a completely new mode for imaging objects with passive radiation with significant potential applications.**
- **ML/EM reconstruction algorithm very recently developed.**
  - Promising results on simulated scenes.
  - Working on understanding issues with experimental data.
  - May need to add regularization.
  - Reconstruction in near real time will be challenging.
- **For yes/no detection of contraband high-Z objects several other analysis methods are proving effective.**
  - Heuristic data reduction methods involving tracing highly scattered rays.
  - Simple featurization of ray data coupled with SVM classification.
  - These methods tested on thousands of simulated cases with varying cargos.
  - We are building a large scale experimental prototype to validate simulated results and demonstrate robust large scale detectors.